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Energy gain of heavy quarks by fluctuations in the QGP

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The collisional energy gain of a heavy quark due to chromo-electromagnetic field fluctuations in a quark-gluon plasma is investigated. The field fluctuations lead to an energy gain of the quark for all temperatures and velocities. The net effect is a reduction of the collisional energy loss by 15-30% for parameters relevant at RHIC energies.

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The aim of the ongoing relativistic heavy-ion collision experiments is to explore the possible plasma phase of QCD, the so called quark-gluon plasma (QGP). High energy partons produced in initial partonic sub-processes in collisions between two heavy nuclei will lose their energy while propagating through the dense matter formed after such collisions, resulting in jet quenching. The amount of quenching depends upon the state of the fireball produced and the resulting quenching pattern may be used for identifying and investigating the plasma phase [1]. In order to quantitatively understand medium modifications of hard parton characteristics in the final state, the energy loss of partons in the QGP has to be determined. There are two contributions to the energy loss of a parton in a QGP: one is caused by elastic collisions between the partons and the other is caused by radiative losses. The radiative loss due to multiple gluon radiation (see [2, 3] for a review) dominates over the collisional one in the ultra-relativistic case. But it has been shown recently that for realistic values of the parameters relevant for heavy-ion collisions, there is a window in which the magnitude of the collisional loss is comparable to the radiative loss for heavy [4, 5, 6, 7] as well as for light [4, 8] quark flavors.

Usually in the calculation of the energy loss the medium is treated in an average manner, *i.e.*, fluctuations are neglected. However, the QGP, being a statistical system, is characterized by omnipresent fluctuations. The fluctuations are not only present at the microscopic level but may also be manifested in the macroscopic level. It is well known that the resulting motion of charged particles in such an environment are stochastic in nature and resemble Brownian motion. Within linear response theory the correlation function of the fluctuations of charge and current densities and the electromagnetic fields in the medium with space-time dispersions are completely determined in terms of the dielectric tensor of the medium. The effect of field fluctuations on the passage of a charged particle through a non-relativistic classical plasma has been worked out by several authors [9, 10, 11, 12, 13, 14] in the past. This effect leads to an energy gain of the particles and is most effective in the low velocity limit. Given the fact that the subject of the energy loss is of topical interest, it is the principal motivation of the present article to quantitatively estimate the effect of fluctuations on the energy loss of a heavy quark passing through an equilibrium, weakly-coupled QGP.

In the semiclassical approach, the collisional energy loss of a heavy quark arises from polarization effects of the medium. It is assumed that the energy lost by the particle per unit time is small compared to the energy of the particle itself, and therefore the change in the velocity of the particle during the motion may be neglected, *i.e*, the particle moves in a straight line trajectory. The energy loss of a particle is determined by the work of the retarding forces acting on the particle in the plasma from the chromo-electric field generated by the particle itself while moving. So the energy loss of the particle per unit time is given by,

$$\frac{dE}{dt} = Q^a \vec{v} \cdot \vec{\mathcal{E}}^a|_{\vec{r}=\vec{v}t} \,, \tag{1}$$

where the field is taken at the location of the particle. In the Abelian approximation, the total chromo-electric field $\vec{\mathcal{E}}^a$ in the QGP can be related to the external current of the test charge by solving Maxwell's equations and the equation of continuity

$$\left[\epsilon_{ij}\left(\omega,k\right) - \frac{k^2}{\omega^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right]\mathcal{E}^a_j\left(\omega,k\right) = \frac{1}{i\omega} j^a_i\left(\omega,k\right) \tag{2}$$

with the color charge current $\vec{j}^a$. In an isotropic and homogeneous plasma the dielectric tensor ϵ_{ij} can be decomposed into longitudinal and transverse parts,

$$\epsilon_{ij}\left(\omega,k\right) = \epsilon_l\left(\omega,k\right)\mathcal{P}^L_{ij} + \epsilon_t\left(\omega,k\right)\mathcal{P}^T_{ij} \,, \tag{3}$$

where, $\mathcal{P}^L_{ij} = k_i k_j/k^2$ and $\mathcal{P}^T_{ij} = \delta_{ij} - k_i k_j/k^2$. The correlation function of the chromo-electromagnetic fields follow from the fluctuation-dissipation theorem and is

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completely determined by the dielectric functions of the medium [9, 10],

$$\left\langle \mathcal{E}_i^a \mathcal{E}_j^a \right\rangle_{\omega,k} = \frac{2}{e^{\beta\omega}-1}\left\{ \mathcal{P}_{ij}^L \frac{\mathrm{Im}\,\epsilon_l}{|\epsilon_l|^2} + \mathcal{P}_{ij}^T \frac{\mathrm{Im}\,\epsilon_t}{|\epsilon_t - \eta^2|^2}\right\} \quad (4)$$

where $\eta = k/\omega$. The gauge invariant high-temperature expression for the dielectric functions are given by (see e.g. [15])

$$\begin{aligned} \epsilon_l(\omega,k) &= 1 + \frac{m_D^2}{k^2}\left[1 - \frac{\omega}{2k}\left(\ln\left|\frac{\omega+k}{\omega-k}\right| \right.\right. \\ &\quad \left.\left. - i\pi\Theta(k^2-\omega^2)\right)\right] , \\ \epsilon_t(\omega,k) &= 1 - \frac{m_D^2}{2\omega^2}\left[\frac{\omega^2}{k^2} + \left(1-\frac{\omega^2}{k^2}\right)\frac{\omega}{2k}\left(\ln\left|\frac{\omega+k}{\omega-k}\right| \right.\right. \\ &\quad \left.\left. - i\pi\Theta(k^2-\omega^2)\right)\right] , \end{aligned} \quad (5)$$

where $m_D^2 = g^2T^2\,(1+N_f/6)$ is the Debye mass squared.

The previous formula for the energy loss in (1) does not take into account the field fluctuation in the plasma and the particle recoil in collisions. To accommodate these effects it is necessary to replace (1) with [9, 10],

$$\frac{dE}{dt} = \left\langle Q^a(t)\,\vec{v}(t)\cdot\vec{\mathcal{E}}^a(\vec{r}(t),t)\right\rangle , \quad (6)$$

where $\langle\cdots\rangle$ denotes the statistical averaging operation [16]. In the classical case, where we can use the concept of trajectory, the equations of motion of the particle have the form [17],

$$\begin{aligned} \frac{d\vec{p}}{dt} &= Q^a(t)\left[\vec{\mathcal{E}}^a(\vec{r}(t),t) + \vec{v}\times\vec{\mathcal{B}}^a(\vec{r}(t),t)\right] , \\ \frac{dQ^a(t)}{dt} &= -g f_{abc} Q^b(t)\, A_\mu^c v^\mu . \end{aligned} \quad (7)$$

Integrating the equations of motion (7) to leading order we find,

$$\begin{aligned} \vec{v}(t) &\simeq \vec{v}_0 + \frac{1}{E_0}\int_0^t dt_1\, Q^a(t_1)\,\vec{\mathcal{F}}^a(\vec{r}(t_1),t_1) , \\ \vec{r}(t) &\simeq r_0 + \frac{1}{E_0}\int_0^t dt_1 \int_0^{t_1} dt_2\, Q^a(t_2)\,\vec{\mathcal{F}}^a(\vec{r}(t_2),t_2) , \end{aligned} \quad (8)$$

$$\begin{aligned} Q^a(t) &= \mathcal{P}\exp\left(-\int_{\vec{r}(t_1)}^{\vec{r}(t)} g f_{abc} A_\mu^b dx^\mu\right) Q^c(t_1) \\ &= U_{ac}(\vec{r}(t),\vec{r}(t_1))\, Q^c(t_1) , \end{aligned} \quad (9)$$

where $\vec{\mathcal{F}} = \vec{\mathcal{E}} + \vec{v}\times\vec{\mathcal{B}}$ and E_0 is the initial parton energy. The mean change in energy of the particle per unit time is given by (6). Let us pick a time interval Δt sufficiently large with respect to the period of random fluctuations of the electric field in the plasma but small compared with the time during which the particle motion changes appreciably. Since the particle trajectory differs only slightly from a straight line during this time interval, the parton velocity and the field acting on the particle at time $t = \Delta t$ can be expanded around the unperturbed (*i.e.* straight line) trajectory. Keeping the leading order terms we get,

$$\begin{aligned} &\vec{v}(t) = \vec{v}_0 + \frac{1}{E_0}\int_0^t dt_1 Q^a(t_1)\,\vec{\mathcal{F}}^a(\vec{r}_0(t_1),t_1) , \\ &\vec{\mathcal{E}}^a(\vec{r}(t),t) = \vec{\mathcal{E}}^a(\vec{r}_0(t),t) + \frac{Q^b(t)}{E_0}\int_0^t dt_1 \int_0^{t_1} dt_2 \\ &\sum_j \mathcal{E}_j^b(\vec{r}_0(t_2),t_2)\,\frac{\partial}{\partial r_{0j}}\,\vec{\mathcal{E}}^a(\vec{r}_0(t),t) . \end{aligned} \quad (10)$$

We substitute (9) and (10) in (6) and assume that the distribution of the color charges of the partons at a given instant t is random and independent of the color fields $\langle Q^a(t)Q^b(t)\rangle = 4\pi C_F\alpha_s$, where C_F is the quadratic Casimir constant in the fundamental representation. Using consistently the Abelian approximation, $U_{ab} = \delta_{ab}$, and keeping in mind that $\langle \mathcal{E}_i^a \mathcal{B}_j^a\rangle = 0$ [18], we get

$$\begin{aligned} \frac{dE}{dt} &= \left\langle Q^a(t)\,\vec{v}_0\cdot\vec{\mathcal{E}}^a(\vec{r}_0(t),t)\right\rangle \\ &+ \frac{C_F\alpha_s}{E_0}\int_0^t dt_1 \left\langle \vec{\mathcal{E}}^a(\vec{r}_0(t_1),t_1)\cdot\vec{\mathcal{E}}^a(\vec{r}_0(t),t)\right\rangle \\ &+ \frac{C_F\alpha_s}{E_0}\int_0^t dt_1 \int_0^{t_1} dt_2 \left\langle \sum_j \mathcal{E}_j^a(\vec{r}_0(t_2),t_2)\times \right. \\ &\quad \left. \frac{\partial}{\partial r_{0j}}\,\vec{v}_0\cdot\vec{\mathcal{E}}^a(\vec{r}_0(t),t)\right\rangle . \end{aligned} \quad (11)$$

Since the mean value of the fluctuating part of the field equals zero, $\left\langle \vec{\mathcal{E}}^a(\vec{r}(t),t)\right\rangle$ equals the chromo-electric field produced by the particle itself in the plasma. The first term in (11) therefore corresponds to the usual polarization loss of the parton calculated in [19]. The second and third terms in (11) correspond to the statistical change in the energy of the moving parton in the plasma due to the fluctuations of the chromo-electromagnetic fields as well as the velocity of the particle under the influence of this field. The part of the energy loss coming form the fluctuations can be summarized as,

$$\begin{aligned} \frac{dE}{dt} &= \frac{C_F\alpha_s}{4\pi^2 E}\int d^3k \left[\frac{\partial}{\partial\omega}\left\langle \omega\vec{\mathcal{E}}_l^2\right\rangle + \left\langle \vec{\mathcal{E}}_t^2\right\rangle\right]_{\omega=\vec{k}\cdot\vec{v}} \\ &= \frac{C_F\alpha_s}{2\pi E v_0^3}\int_0^{k_{\max}v_0} d\omega\, \coth\frac{\beta\omega}{2} F(\omega,k)_{\omega,\frac{\omega}{v_0}} \\ &+ \frac{C_F\alpha_s}{2\pi E v_0}\int_0^{k_{\max}} dk\, k \int_0^{k v_0} d\omega\, \coth\frac{\beta\omega}{2} G(\omega,k) , \end{aligned} \quad (12)$$

where $F(\omega,k) = 2\omega^2\mathrm{Im}\,\epsilon_l/|\epsilon_l|^2$ and $G(\omega,k) = 4\mathrm{Im}\,\epsilon_t/|\epsilon_t - k^2/\omega^2|^2$, and we have taken $E = E_0$ to be the initial energy of the parton. The above expression gives the mean energy (per unit time) absorbed by a propagating particle from the heat bath. Physically, this arises from gluon absorption. Thermal absorption of gluons was also shown to reduce the radiative energy loss [20]. We arrive at a somewhat similar conclusion as there, albeit in a different context. It is to be noted here that since the spectral density of field fluctuations $\left\langle \vec{\mathcal{E}}^2 \right\rangle$ are positive for positive frequencies by definition, according to (12) the particle energy will grow due to interactions with the fluctuating fields. The contribution from field

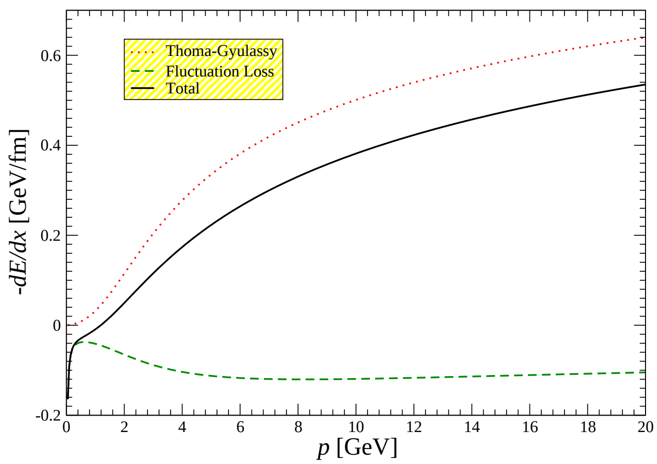

FIG. 1: Various contributions to the energy loss of a charm quark in the QGP. The dotted line corresponds to the collisional energy loss calculated in [19], the dashed line to the energy gain by fluctuations, and the solid line to the sum of the both contributions.

fluctuations to the heavy quark energy loss is shown in Fig. 1 and Fig. 2. Our choice of parameters are $N_f = 2$, $T = 250$ MeV, $\alpha_s = 0.3$, and we take the charm quark mass to be $m_c = 1.5$ GeV. For the upper cut-off $k_{\max}$ we take [21],

$$k_{\max} = \min\left\{E, \frac{2q\,(E+p)}{\sqrt{m_c^2 + 2q\,(E+p)}}\right\}. \tag{13}$$

In Fig. 2 we show the fractional collisional energy loss of the charm where the effect of field fluctuations is taken into account. It is evident that the effect of the fluctuations on the heavy quark energy loss is significant at low momenta. For momenta $5-20$ GeV the fluctuation effect reduces the collisional loss by $15-30\%$. At higher momenta the relative importance of the fluctuation gain to the collisional loss decreases gradually.

Let us note here that the assumption of an equilibrium condition necessary implies isotropization in momentum space. On the other hand, matter created in non-central heavy-ion collisions is anisotropic to start with and the strong longitudinal expansion afterwards, at its own, brings anisotropy in the system [22]. The characteristic feature of such anisotropic systems is the presence of a Weibel-type of instabilities [23, 24, 25]. It has been argued that assuming a turbulent, weakly coupled anisotropic QGP may provide a natural explanation

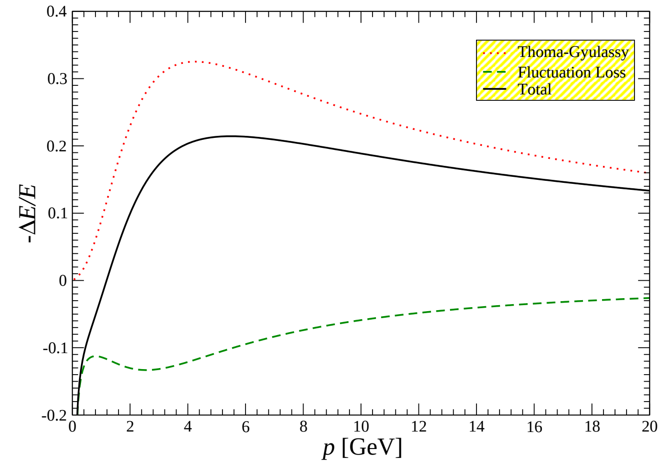

FIG. 2: Relative importance of the fluctuation loss compared to the collisional energy loss of Ref.[19]. We take the path length to be $L = 5$ fm.

for the observed rapid isotropization time [26], for the small shear viscosity to entropy density ratio [27], or for dihadron correlation functions [28].

The analysis presented above can in principle be extended to the case of a non-equilibrium, anisotropic QGP, provided the power spectrum of the electromagnetic fluctuations are known. One possible way is to simulate them on the lattice [29]. Fluctuations are much stronger in non-equilibrium situations than in thermal systems [30]. The effect of field fluctuations on the passage of a heavy quark is therefore expected to be stronger in an anisotropic QGP [31]. Interestingly, the energy loss for a heavy quark in an anisotropic QGP without taking field fluctuations into account is negative (corresponding to energy gain) at low energies similar to our case [32]. In the case of an anisotropic plasma there could be an "anti-Landau" damping mechanism which would lead to an energy loss by fluctuations.

Recently heavy quark probes at RHIC have posed new challenges to the theoretical understanding of parton energy loss. As shown by Wicks et al [6] recent measurement of the non-photonic single electron data cannot be explained by the radiative loss alone. If the collisional energy loss is included, the agreement is better but not satisfactory. On the other hand, using the transport coefficients within pQCD energy loss calculations [33] the elliptic flow coefficient of single electrons v_2 is limited only to $2-3\%$ in semicentral Au-Au collisions, while the experimental values [34] reach up to 10% around transverse electron momenta of 2 GeV/c. It is suggested recently in Ref. [35] that not only the energy loss but also the energy gain in low momenta may be required for obtaining larger theoretical v_2 values. It will be interesting to find out whether the inclusion of the fluctuation gain or loss can shed light on this puzzle.

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